

Binomial Distribution 4

1	$\mu = 300 \times 0.072 = 21.6, \sigma^2 = 20.0448$ $P(x < 18) = P\left(z < \frac{17.5 - 21.6}{\sqrt{20.0448}}\right)$ $= P(z < -0.9157)$ $= 1 - 0.8201$ $= 0.180$	B1 M1 M1 M1 A1	300×0.072 seen and 300×0.072×0.928 seen or implied (σ = 4.4771, σ ² = 20(.0)) oe ±Standardising, their mean/var, with sq root Cont corr 17.5 or 18.5 Correct area 1 - Φ Answer wrt 0.180, nfw
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2	(i) constant / given p , independent trials, fixed / given no. of trials, only two outcomes	B1 B1	Any one correct Any 3 correct
	(ii) $P(x \geq 3) = 1 - P(0, 1, 2)$ $= 1 - [(0.85)^{18} + (0.85)^{17}(0.15) \times 18 +$ $(0.85)^{16}(0.15)^2 \times {}^{18}C_2]$ $= 0.520$	M1 M1 A1	Any binomial expression $p^r(1-p)^{18-r} {}^{18}C_r$ seen $1 - P(0, 1, 2)$, any n, p, q Correct answer

3	(i) $X \sim \text{Bin}(12, 0.2)$	B1 B1 B1	Bin or B 12 0.2 or 1/5
	(ii) $P(X = 3, 4, 5) = 0.2^3 0.8^9 {}_{12}C_3 + 0.2^4 0.8^8 {}_{12}C_4$ $+ 0.2^5 0.8^7 {}_{12}C_5$ $= 0.23622 + 0.13287 + 0.05315$ $= 0.422$	M1 A1ft A1	Bin expression with any p Correct unsimplified expression, their p Correct answer

	(iii) $P(X = 0) < 0.01$ $0.8^n < 0.01$ $n = 21$	M1 M1 A1	[3] Statement involving $P(X = 0)$ and 0.01 can be implied Equn involving '0.8', 0.01 or 0.99 Correct answer
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4	(i)	$p = 4/9$ or $5/9$	B1	[3]	Binomial term ${}_5C_x p^x (1-p)^{5-x}$ seen
		$P(\text{at least } 2) = 1 - P(0, 1)$ $= 1 - (5/9)^5 - (4/9)(5/9)^4 {}_5C_1$	M1		
	$= 0.735$	A1	Correct answer		
	(ii)	$np = 96$ $npq = 32$ $p = P(\leq k)$	M1		Using $np = 96$ $npq = 32$ to obtain eqn in 1 variable
		$p = 2/3$ $q = 1/3$ $n = 144$ $k = 6$	A1 A1 ft		1/3 or 2/3 seen or implied Correct k ft $k = 9p$
		$n = 144$	A1	[4]	correct n

5	(i)	$(0.8)^n < 0.001$	M1	[3]	Eqn or inequ involving 0.8^n or 0.2^n and 0.001 or 0.999 Trial and error or logs (can be implied) Correct answer MR 0.01, max available M1M1A0
		$n > 30.9$	M1		
		$n = 31$	A1		
	(ii)	$\mu = 120 \times 0.2 = 24$ $\sigma^2 = 120 \times 0.2 \times 0.8 = 19.2$ $P(x < 33) = P\left(z < \frac{32.5 - 24}{\sqrt{19.2}}\right)$ $= P(z < 1.9398)$ $= 0.974$	B1 M1 M1		
		A1	[4]	Correct answer	

6	(i)	$np = 24$, $npq = 4.8$	B1	[4]	24 and 4.8 or $\sqrt{4.8}$ seen can be unsimplified Standardising, need sq rt, cc not necessary Continuity correction 24.5 or 25.5 used Correct answer must be from 24.5
		$z = \pm \left(\frac{24.5 - 24}{\sqrt{4.8}}\right) = 0.228$	M1 M1		
		Prob = 0.590	A1		
	(ii)	np and nq both > 5 .	B1		[1]

<p>7 (i) $P(X < 3) = P(0) + P(1) + P(2)$ $= (0.84)^{11} + (0.16)(0.84)^{10} \times {}^{11}C_1 +$ $(0.16)^2(0.84)^9 \times {}^{11}C_2$ $= 0.1469 + 0.30782 + 0.2931$ $= 0.748$</p>	<p>M1 M1 A1 [3]</p>	<p>Binomial term with ${}^{11}C_r p^r (1-p)^{11-r}$ seen Correct expression for $P(0, 1, 2)$ or $P(0, 1, 2, 3)$ Can have wrong p Correct final answer. Normal approx M0 M0 A0</p>
<p>(ii) $\mu = 125 \times 0.64 = 80$ $\sigma^2 = 125 \times 0.64 \times 0.36 = 28.8$ $P(X > 73) = 1 - \Phi\left(\frac{73.5 - 80}{\sqrt{28.8}}\right)$ $= \Phi(1.211)$ $= 0.887$</p>	<p>B1 M1 M1 M1 A1 [5]</p>	<p>80 and 28.8 or 5.37 seen standardising, with or without cc, must have sq rt in denom continuity correction 73.5 or 72.5 only correct region (> 0.5 if mean > 73.5, vv if mean < 73.5) correct answer</p>

<p>8 (i)</p>	<p>$P(\geq 3) = 1 - P(0, 1, 2)$ $= 1 - (6/7)^{15} - {}_{15}C_1 (1/7) (6/7)^{14} - {}_{15}C_2 (1/7)^2 (6/7)^{13}$ $(= 1 - 0.0990 - 0.2476 - 0.2889)$ $= 0.365$ (accept 0.364)</p>	<p>M1 M1 A1 A1 4</p>	<p>For attempt at $1 - P(0, 1, 2)$ or $1 - P(0, 1, 2, 3)$ or $P(3...15)$ or $P(4...15)$ For 1 or more terms with $1/7$ and $6/7$ to powers which sum to 15 and ${}_{15}C_{\text{something}}$ Completely correct unsimplified form Correct final answer</p>
<p>(ii)</p>	<p>$\mu = 56 \times 1/7 (= 8)$ $\sigma^2 = 56 \times 1/7 \times 6/7 (= 6.857)$ $P(\text{more than } 7) = 1 - \Phi\left(\frac{7.5 - 8}{\sqrt{6.857}}\right)$ $= \Phi\left(\frac{8 - 7.5}{\sqrt{6.857}}\right) = \Phi(0.1909)$ $= 0.576$</p>	<p>B1 M1 M1 M1 A1 5</p>	<p>8 and 6.857 or 6.86 or 2.618 seen or implied Standardising attempt with or without cc, must have square root Continuity correction either 7.5 or 6.5 Final answer > 0.5 (award this if the long way is used and the final answer is > 0.5) Correct final answer</p>

<p>9 (i) $P(0) = (0.8)^{15} (= 0.03518)$ $P(1) = {}_{15}C_1 \times (0.2) \times (0.8)^{14}$ $(= 0.1319)$ $P(2) = {}_{15}C_2 \times (0.2)^2 \times (0.8)^{13}$ $(= 0.2309)$</p> <p>$P(X \leq 2) = 0.398$</p>	<p>B1 B1 B1 3</p>	<p>For correct numerical expression for P(0) For correct numerical expression for P(1) or P(2) For answer rounding to 0.398</p>
<p>(ii) $1 - (0.8)^n \geq 0.85$ $0.15 \geq (0.8)^n$</p> <p>$n = 9$</p>	<p>M1 M1 dep A1 3</p>	<p>For an equality/inequality involving 0.8, n, 0.85 For solving attempt (could be trial and error or lg) For correct answer</p>
<p>(iii) $\mu = 1600 \times 0.2 = 320,$ $\sigma^2 = 1600 \times 0.2 \times 0.8 = 256$ $P(X \geq 290) \text{ or } P(X < 350)$ $= 1 - \Phi\left(\frac{289.5 - 320}{\sqrt{256}}\right) = 1 - \Phi(-1.906)$</p> <p>$= \Phi(1.906) = 0.972$</p>	<p>B1 M1 M1 M1 A1 5</p>	<p>For both mean and variance correct For standardising, with or without cc, must have $\sqrt{\quad}$ on denom For use of continuity correction 289.5 or 290.5 For finding an area > 0.5 from their z For answer rounding to 0.972</p>