

Differential Equations MS 1

- 1 (i) Separate variables correctly and attempt to integrate at least one side B1
 Obtain term $\ln R$ B1
 Obtain $\ln x - 0.57x$ B1
 Evaluate a constant or use limits $x = 0.5, R = 16.8$, in a solution containing terms of the form $a \ln R$ and $b \ln x$ M1
 Obtain correct solution in any form A1
 Obtain a correct expression for R , e.g. $R = xe^{(3.80 - 0.57x)}, R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$ A1 [6]
- (ii) Equate $\frac{dR}{dx}$ to zero and solve for x M1
 State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75 A1
 Obtain $R = 28.8$ (allow 28.9) A1 [3]
- 2 (i) Sensibly separate variables and attempt integration of at least one side M1
 Obtain $2y^{\frac{1}{2}} = \dots$ or equivalent A1
 Correct integration by parts of $x \sin \frac{1}{3}x$ as far as $ax \cos \frac{1}{3}x \pm \int b \cos \frac{1}{3}x dx$ M1
 Obtain $-3x \cos \frac{1}{3}x + \int 3 \cos \frac{1}{3}x dx$ or equivalent A1
 Obtain $-3x \cos \frac{1}{3}x + 9 \sin \frac{1}{3}x$ or equivalent A1
 Obtain $y = \left(-\frac{3}{10}x \cos \frac{1}{3}x + \frac{9}{10} \sin \frac{1}{3}x + c \right)^2$ or equivalent A1 [6]
- (ii) Use $x = 0$ and $y = 100$ to find constant M*1
 Substitute 25 and calculate value of y DM*1
 Obtain 203 A1 [3]
- 3 Separate variables and integrate one side B1
 Obtain term $\ln(x + 2)$ B1
 Use $\cos 2A$ formula to express $\sin^2 2\theta$ in the form $a + b \cos 4\theta$ M1
 Obtain correct form $(1 - \cos 4\theta)/2$, or equivalent A1
 Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent A1[√]
 Evaluate a constant, or use $\theta = 0, x = 0$ as limits in a solution containing terms $c \ln(x + 2), d \sin(4\theta), e\theta$ M1
 Obtain correct solution in any form, e.g. $\ln(x + 2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$ A1
 Use correct method for solving an equation of the form $\ln(x + 2) = f$ M1
 Obtain answer $x = 0.962$ A1 [9]

- 4 (i) State $\frac{dN}{dt} = k(N - 150)$ **B1** [1]
- (ii) Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k **M1**
 Obtain $k = 0.08$ **A1**
 Separate variables and obtain general solution involving $\ln(N - 150)$ **M1***
 Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$ **A1[√]**
 Substitute $t = 0$ and $N = 650$ to find c **dep M1***
 Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent **A1**
 Obtain $N = 500e^{0.08t} + 150$ **A1** [7]
- (iii) Either Substitute $t = 15$ to find N or solve for t with $N = 2000$ **M1**
 Obtain Either $N = 1810$ or $t = 16.4$ and conclude target not met **A1** [2]