Differential Equations QP 1

1 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{dR}{dx} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

2 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

3 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x + 2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

4 Naturalists are managing a wildlife reserve to increase the number of plants of a rare species. The number of plants at time t years is denoted by N, where N is treated as a continuous variable.

(i) It is given that the rate of increase of N with respect to t is proportional to $(N - 150)$. Write down a differential equation relating N, t and a constant of proportionality. [1]

(ii) Initially, when $t = 0$, the number of plants was 650. It was noted that, at a time when there were 900 plants, the number of plants was increasing at a rate of 60 per year. Express N in terms of t. [7]

(iii) The naturalists had a target of increasing the number of plants from 650 to 2000 within 15 years. Will this target be met? [2]