Complex Numbers MS 1

1 **(i)** Substitute $z = 1 + \mathrm{i}$ and obtain $w = \dfrac{1+2\mathrm{i}}{1+\mathrm{i}}$ B1

EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent M1

Simplify numerator to $3 + \mathrm{i}$ or denominator to 2 A1

Obtain final answer $\frac{3}{2} + \frac{1}{2}\mathrm{i}$, or equivalent A1

OR: Obtain two equations in x and y, and solve for x or for y M1

Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent A1

Obtain final answer $\frac{3}{2} + \frac{1}{2}\mathrm{i}$, or equivalent A1 **[4]**

(ii) *EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$ B1

Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y M1

OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts B1

Solve for x and y M1

Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$ A1

Obtain final answer $-\frac{\sqrt{3}}{2} + \frac{1}{2}\mathrm{i}$ A1 **[4]**

2 **(i)** State or imply $\mathrm{i}w = -3 + 5\mathrm{i}$ B1

Carry out multiplication by $\dfrac{4-\mathrm{i}}{4-\mathrm{i}}$ M1

Obtain final answer $-\frac{7}{17} + \frac{23}{17}\mathrm{i}$ or equivalent A1 [3]

(ii) Multiply w by z to obtain $17 + 17\mathrm{i}$ B1

State $\arg w = \tan^{-1}\frac{3}{5}$ or $\arg z = \tan^{-1}\frac{1}{4}$ B1

State $\arg wz = \arg w + \arg z$ M1

Confirm given result $\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4} = \frac{1}{4}\pi$ legitimately A1 [4]

3 **(i)** Show u in a relatively correct position — **B1**
Show u^* in a relatively correct position — **B1**
Show $u^* - u$ in a relatively correct position — **B1**
State or imply that $OABC$ is a parallelogram — **B1** [4]

(ii) *EITHER*: Substitute for u and multiply numerator and denominator by $3 + i$, or equivalent — **M1**
Simplify the numerator to $8 + 6i$ or the denominator to 10 — **A1**
Obtain final answer $\frac{4}{5} + \frac{3}{5}i$, or equivalent — **A1**
OR: Substitute for u, obtain two equations in x and y and solve for x or for y — **M1**
Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent — **A1**
Obtain final answer $\frac{4}{5} + \frac{3}{5}i$, or equivalent — **A1** [3]

(iii) State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$ — **B1**
Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$ — **M1**
Fully justify the given statement using exact values — **A1** [3]

4 **(a)** EITHER: Use quadratic formula to solve for w — **M1**
Use $i^2 = -1$ — **M1**
Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$ — **A1**
Multiply numerator and denominator of an answer by $-2i + 1$, or equivalent — **M1**
Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ — **A1**

OR1: Multiply the equation by $1 - 2i$ — **M1**
Use $i^2 = -1$ — **M1**
Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent — **A1**
Use quadratic formula or factorise to solve for w — **M1**
Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ — **A1**

OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts — **M1**
Use $i^2 = -1$ — **M1**
Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e. — **A1**
Form equation in x only or y only and solve — **M1**
Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ — **A1** [5]

(b) Show a circle with centre $1 + i$ — **B1**
Show a circle with radius 2 — **B1**
Show half-line $\arg z = \frac{1}{4}\pi$ — **B1**
Show half-line $\arg z = -\frac{1}{4}\pi$ — **B1**
Shade the correct region — **B1** [5]

5	(i)	State modulus $2\sqrt{2}$, or equivalent	B1	
		State argument $-\frac{1}{3}\pi$ (or $-60°$)	B1	[2]
	(ii) (a)	State answer $3\sqrt{2}+\sqrt{6}\,\mathrm{i}$	B1	
	(b)	*EITHER:* Substitute for z and multiply numerator and denominator by conjugate of iz	M1	
		Simplify the numerator to $4\sqrt{3}+4\,\mathrm{i}$ or the denominator to 8	A1	
		Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	A1	
		OR: Substitute for z, obtain two equations in x and y and solve for x or for y	M1	
		Obtain $x=\frac{1}{2}\sqrt{3}$ or $y=\frac{1}{2}$	A1	
		Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	A1	[4]
	(iii)	Show points A and B in relatively correct positions	B1	
		Carry out a complete method for finding angle AOB, e.g. calculate the argument of $\frac{z^*}{\mathrm{i}z}$	M1	
		Obtain the given answer	A1	[3]