Complex Numbers QP 1

1 The complex numbers w and z satisfy the relation

$$w = \frac{z + \mathrm{i}}{\mathrm{i}z + 2}.$$

(i) Given that $z = 1 + \mathrm{i}$, find w, giving your answer in the form $x + \mathrm{i}y$, where x and y are real. [4]

(ii) Given instead that $w = z$ and the real part of z is negative, find z, giving your answer in the form $x + \mathrm{i}y$, where x and y are real. [4]

2 The complex numbers w and z are defined by $w = 5 + 3\mathrm{i}$ and $z = 4 + \mathrm{i}$.

(i) Express $\frac{\mathrm{i}w}{z}$ in the form $x + \mathrm{i}y$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]

3 The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\frac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

4 **(a)** Solve the equation $(1 + 2\mathrm{i})w^2 + 4w - (1 - 2\mathrm{i}) = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - \mathrm{i}| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

5 The complex number z is defined by $z = (\sqrt{2}) - (\sqrt{6})\mathrm{i}$. The complex conjugate of z is denoted by z^*.

(i) Find the modulus and argument of z. [2]

(ii) Express each of the following in the form $x + \mathrm{i}y$, where x and y are real and exact:

(a) $z + 2z^*$;

(b) $\dfrac{z^*}{\mathrm{i}z}$.

[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z^* and $\mathrm{i}z$ respectively. Prove that angle AOB is equal to $\frac{1}{6}\pi$. [3]