Vectors MS 1

1 (i)	*EITHER*:	Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i} - 17\mathbf{j} + 4\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	B1	
		Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1	
		Solve and obtain $\lambda = 3$	A1	
		Carry out a complete method for finding the length of AP	M1	
		Obtain the given answer 15 correctly	A1	
	*OR*1:	Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}$	B1	
		Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	M1	
		Obtain correct answer, e.g. $-30\mathbf{i} + 6\mathbf{j} + 33\mathbf{k}$	A1	
		Divide the modulus of the product by that of the direction vector	M1	
		Obtain the given answer correctly	A1	
	*OR*2:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
		Use a scalar product to find the projection of BA (or AB) on l	M1	
		Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}}$	A1	
		Use Pythagoras to find the perpendicular	M1	
		Obtain the given answer correctly	A1	
	*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
		Use a scalar product to find the cosine of ABP	M1	
		Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}.\sqrt{306}}$	A1	
		Use trig. to find the perpendicular	M1	
		Obtain the given answer correctly	A1	
	*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
		Find a second point C on l and use the cosine rule in triangle ABC to find the cosine of angle A, B, or C, or use a vector product to find the area of ABC	M1	
		Obtain correct answer in any form	A1	
		Use trig. or area formula to find the perpendicular	M1	
		Obtain the given answer correctly	A1	
	*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ in any form	B1	
		Use correct method to express AP^2 (or AP) in terms of λ	M1	
		Obtain a correct expression in any form, e.g. $(1-2\lambda)^2 + (-17+\lambda)^2 + (4-2\lambda)^2$	A1	
		Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
		Obtain the given answer correctly	A1	**[5]**

2	(i)	State at least two of the equations $1+\lambda=a+\mu$, $4=2+2\mu$, $-2+3\lambda=-2+3a\mu$	B1	
		Solve for λ or for μ	M1	
		Obtain $\lambda=a$ (or $\lambda=a+\mu-1$) and $\mu=1$	A1	
		Confirm values satisfy third equation	A1	[4]
	(ii)	State or imply point of intersection is $(a+1, 4, 3a-2)$	B1	
		Use correct method for the modulus of the position vector and equate to 9, following their point of intersection	M*1	
		Solve a three-term quadratic equation in a $(a^2-a-6=0)$	DM*1	
		Obtain –2 and 3	A1	[4]
3	(i)	Use correct method to form a vector equation for *AB*	**M1**	
		Obtain a correct equation, e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}+\lambda(2\mathbf{i}-2\mathbf{j}+\mathbf{k})$ or $\mathbf{r}=3\mathbf{i}+\mathbf{k}+\mu(2\mathbf{i}-2\mathbf{j}+\mathbf{k})$	**A1**	[2]
	(ii)	Using a direction vector for *AB* and a relevant point, obtain an equation for *m* in any form	**M1**	
		Obtain answer $2x-2y+z=4$, or equivalent	**A1**	[2]
	iii)	Express general point of *AB* in component form, e.g. $(1+2\lambda, 2-2\lambda, \lambda)$ or $(3+2\mu, -2\mu, 1+\mu)$	**B1**√	
		Substitute in equation of *m* and solve for λ or for μ	**M1**	
		Obtain final answer $\frac{7}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$ for the position vector of *N*, from $\lambda=\frac{2}{3}$ or $\mu=-\frac{1}{3}$	**A1**	
		Carry out a correct method for finding *CN*	**M1**	
		Obtain the given answer $\sqrt{13}$	**A1**	[5]
		[The f.t. is on the direction vector for *AB*.]		

4	(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$	B1	
		Use correct method to calculate their scalar product	M1	
		Show value is zero and planes are perpendicular	A1	[3]
	(ii)	*EITHER*: Carry out a complete strategy for finding a point on l the line of intersection	M1	
		Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0)	A1	
		EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l, e.g. $3a+b-c=0$ and $a-b+2c=0$	B1	
		Solve for one ratio, e.g. $a:b$	M1	
		Obtain $a:b:c=1:-7:-4$, or equivalent	A1	
		State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	A1√	
		*OR*1: Obtain a second point on l, e.g. (1, 0, 1)	B1	
		Subtract vectors and obtain a direction vector for l	M1	
		Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent	A1	
		State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$	A1√	
		*OR*2: Attempt to find the vector product of the two normal vectors	M1	
		Obtain two correct components of the product	A1	
		Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent	A1	
		State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	A1√	
		*OR*1: Express one variable in terms of a second variable	M1	
		Obtain a correct simplified expression, e.g. $y=7-7x$	A1	
		Express the third variable in terms of the second	M1	
		Obtain a correct simplified expression, e.g. $z=5-4x$	A1	
		Form a vector equation for the line	M1	
		Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	A1√	
		*OR*2: Express one variable in terms of a second variable	M1	
		Obtain a correct simplified expression, e.g. $z=5-4x$	A1	
		Express the same variable in terms of the third	M1	
		Obtain a correct simplified expression e.g. $z=(7+4y)/7$	A1	
		Form a vector equation for the line	M1	
		Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	A1√	[6]

5	(i)	Express general point of l in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$	B1	
		Using the correct process for the modulus form an equation in λ	M1*	
		Reduce the equation to a quadratic, e.g. $6\lambda^2+2\lambda-4=0$	A1	
		Solve for λ (usual requirements for solution of a quadratic)	DM1	
		Obtain final answers $-\mathbf{i}+3\mathbf{j}$ and $\frac{7}{3}\mathbf{i}+\frac{4}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$	A1	[5]
	(ii)	Using the correct process, find the scalar product of a direction vector for l and a normal for p	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$	M1	
		State a correct equation in any form, e.g. $\dfrac{2a-1+1}{\sqrt{(a^2+1+1)}\sqrt{(2^2+(-1)^2+1)}}=\pm\dfrac{2}{3}$	A1	
		Solve for a^2	M1	
		Obtain answer $a=\pm 2$	A1	[5]