Vectors QP 1

1 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

(i) Show that the length of the perpendicular from A to l is 15. [5]

(ii) The line l lies in the plane with equation $ax + by - 3z + 1 = 0$, where a and b are constants. Find the values of a and b. [5]

2 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]

3 The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

(i) Find a vector equation for the line passing through A and B. [2]

(ii) Obtain the equation of the plane m, giving your answer in the form $ax + by + cz = d$. [2]

(iii) The line through A and B intersects the plane m at the point N. Find the position vector of N and show that $CN = \sqrt{(13)}$. [5]

4 Two planes have equations $3x + y - z = 2$ and $x - y + 2z = 3$.

(i) Show that the planes are perpendicular. [3]

(ii) Find a vector equation for the line of intersection of the two planes. [6]

5 The line l has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$.

(i) Find the position vectors of the two points on the line whose distance from the origin is $\sqrt{(10)}$. [5]

(ii) The plane p has equation $ax + y + z = 5$, where a is a constant. The acute angle between the line l and the plane p is equal to $\sin^{-1}(\frac{2}{3})$. Find the possible values of a. [5]