1 A toy for cats consists of a plastic mouse of mass m attached to a spring. When the mouse is on a low-friction horizontal surface, with the spring attached to a rigid support as shown, it performs simple harmonic motion when given a small displacement x from its equilibrium position and released.

(a) Show that the acceleration of the mouse, a, is given by $a = -\left(\dfrac{k}{m}\right)x$, where k is the stiffness of the spring.

(2)

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(b) The mouse has a mass $m = 0.15$ kg and the spring extends by 20 cm when the mouse is supported vertically by the spring.

Calculate the frequency of oscillation of the mouse if it is set into oscillation on a low friction horizontal surface.

(5)

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Frequency = ...

(Total for Question 1 = 7 marks)

2 Read this passage and answer the questions that follow.

The Millennium Bridge opened on 10 June 2000 as London's first new Thames crossing in more than 100 years. The bridge uses "lateral suspension" – an engineering innovation that allows suspension bridges to be built without tall supporting columns. Tens of thousands of people crossed the bridge on its opening day. The structure was designed to take the weight, but suddenly the bridge began to sway and twist in regular oscillations. The worst of the movement occurred on the central span where the edge of the bridge oscillated through a total distance of 70 mm.

To solve the problem the engineers decided to use damping mechanisms – giant shock absorbers to limit the bridge's response to external forces. They decided to use two systems: viscous dampers, similar to car shock absorbers, and tuned mass dampers. A tuned mass damper is a large mass stiffened by springs.

(a) Name the effect that results in a system being driven into large amplitude oscillations, and state the condition necessary for this to happen.
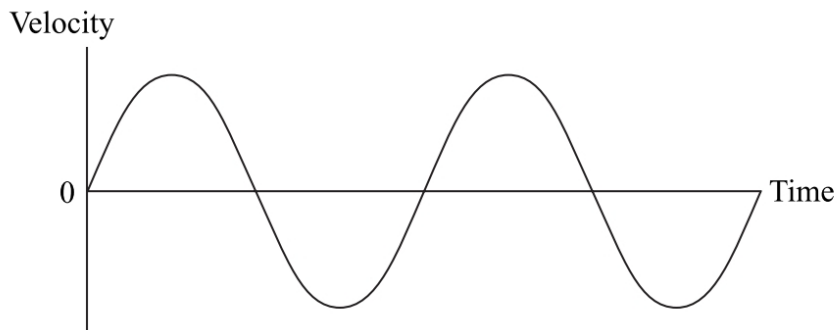
(2)

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(b) The graph shows the variation of velocity with time at the edge of the central span of the bridge.

Velocity

0 ⟋ Time

Mark on this graph:

(i)   An instant X at which the displacement was a maximum.

(1)

(ii) An instant Y at which the acceleration was zero.

(1)

(c) Before modification the edge of the central span of the bridge oscillated with simple harmonic motion, and had a maximum acceleration of 0.89 m s^{-2}. Calculate the maximum velocity of the edge of the central span of the bridge.

(4)

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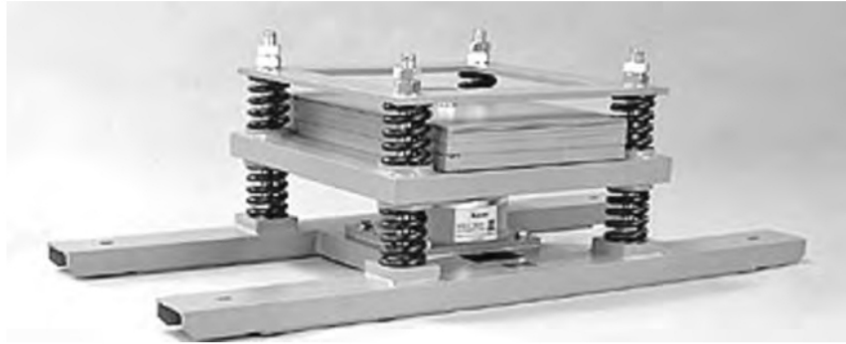
Maximum velocity = ...

(d) The photograph shows the tuned mass dampers which were fitted to the bridge.
They are tuned to the natural frequency of oscillation of the bridge.

Discuss how the tuned mass dampers reduce the amplitude of the oscillations of the bridge and explain why they must be very heavily damped.

(3)

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(Total for Question   = 11 marks)

3 Observing the display of a 'floating image' clock relies on the phenomenon of 'persistence of vision'. The clock has a wand with a set of flashing light-emitting diodes (LEDs) at its end. The wand oscillates rapidly back and forth and takes only 0.0625 s to sweep from one end to the other. The wand becomes almost invisible to the eye, while the flashing LEDs create a floating image effect.

Flashing LED
Wand
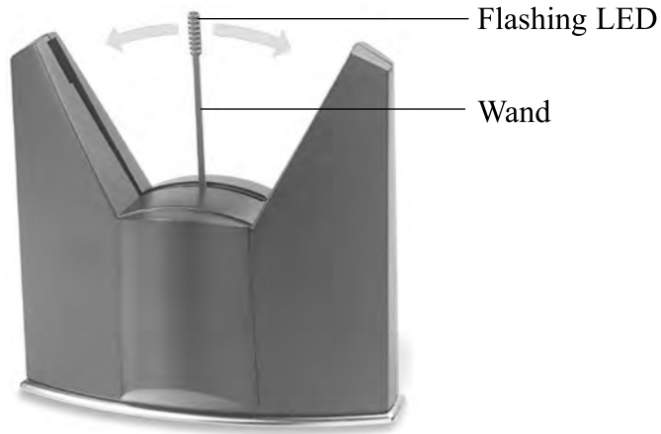
(a) The tip of the wand moves with simple harmonic motion as it sweeps through a distance of 10.0 cm from one end to the other.

(i) Calculate the frequency of the wand's oscillation.

(2)

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Frequency = ...

(ii) The speed of the wand varies as it sweeps back and forth. At what point will the speed of the wand be a maximum?

(1)

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(iii) Calculate the maximum speed of the tip of the wand.

(2)

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Maximum speed = ...

(b) In normal operation the clock may make a faint ticking or humming sound. An unstable surface supporting the clock can result in noisy operation due to resonance.

(i) Explain what is meant by resonance.

(2)

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(ii) The clock is mounted on rubber feet so that it does not make direct contact with surfaces. Explain how this helps to reduce the effects of resonance.
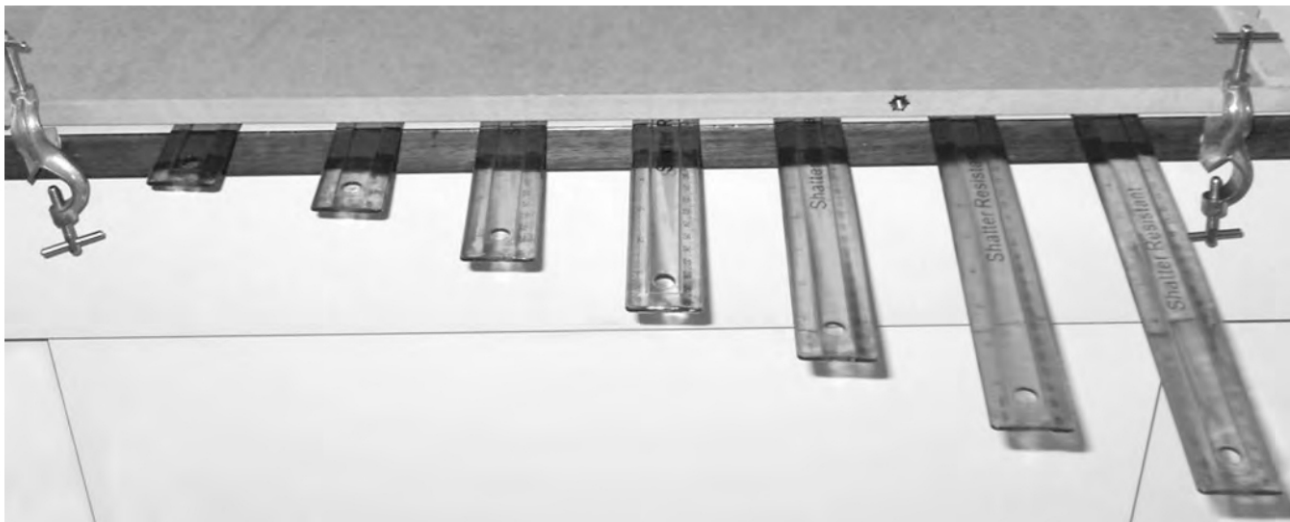
(2)

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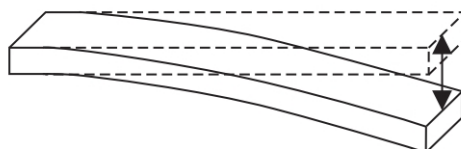
(Total for Question   = 9 marks)

4 A student makes the "ruler piano" shown in the photograph.

One end of each ruler is held flat on the desk whilst the other end is set into oscillation. Each ruler oscillates at a different frequency. Some of the rulers produce an audible sound.

(a) State the condition for an oscillation to be simple harmonic.

(2)

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(b) The end of one ruler moves through 5.0 cm from one extreme position to the other, and makes 10 complete oscillations in 4.5 s.

Calculate the maximum velocity of this end.

(3)

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Maximum velocity = ...

(c) A standing wave is set up on each oscillating ruler.

Explain why each length of ruler oscillates at a different frequency.

(3)

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(Total for Question   = 8 marks)

5 (a) Define simple harmonic motion.

(2)

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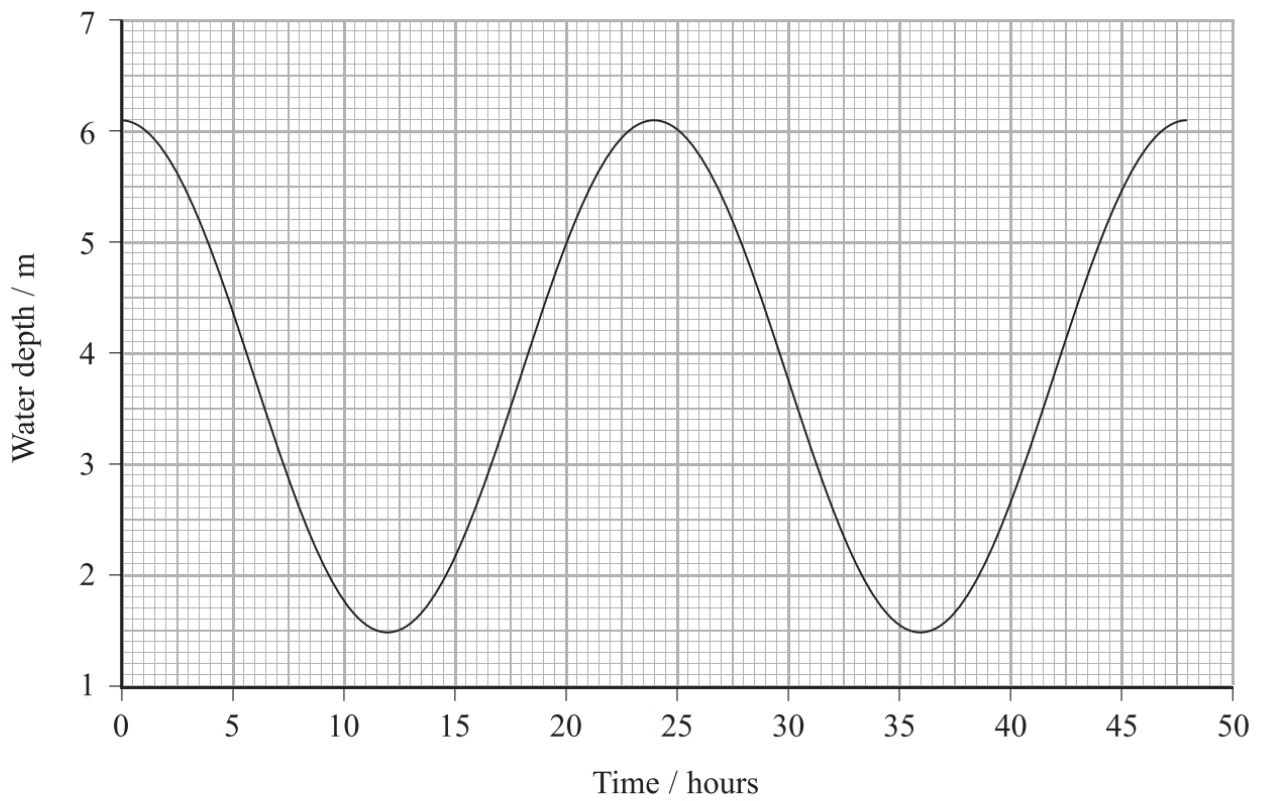
(b) The graph shows the variation in water level displacement with time for the water in a harbour. The water level displacement varies with simple harmonic motion.

(i) Use the graph to calculate the amplitude and the time period of the variation in the water level displacement.

(2)

Amplitude = ...

Time period = ...

(ii) Show that the maximum rate of change of water level displacement is about 0.6 m hour^{-1}.

(3)

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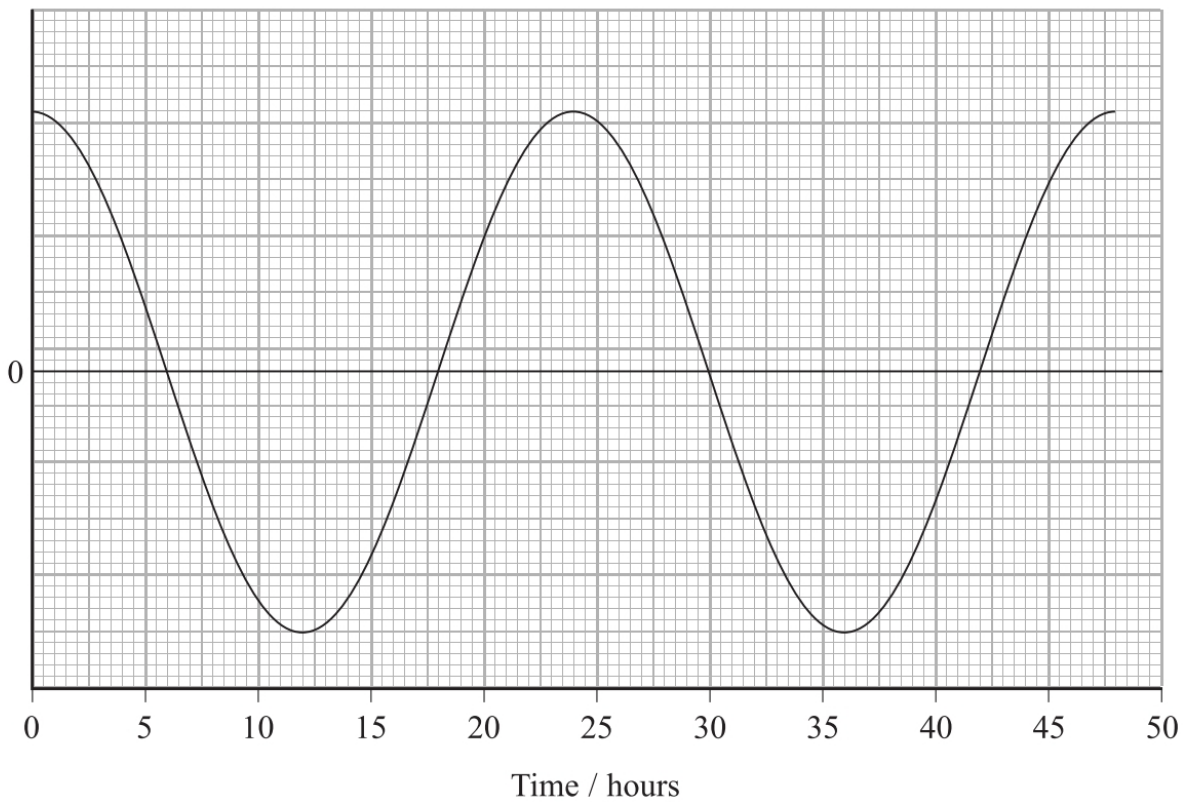
(iii) On the axis below sketch how the rate of change of water level displacement varies with time for the interval 0–30 hours. The variation in water level displacement with time has been drawn for you. You need not add any numerical values to the y-axis.

(2)

Time / hours

(Total for Question   = 9 marks)

6 The photograph shows a nodding tiger toy. The tiger is placed on a car's dashboard and its head nods up and down as the car is driven along a rough road surface.

It is noticed that at a particular speed the tiger's head vibrates with maximum amplitude.

(a) (i)   What is the name of this phenomenon?

(1)

....

(ii) Describe the conditions necessary for this phenomenon to occur.

(2)

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(b) (i) The graph shows the variation of acceleration with time for the tiger's head. Using values from the graph calculate the amplitude of oscillation of the tiger's head.

(3)

Acceleration/m s^{-2}

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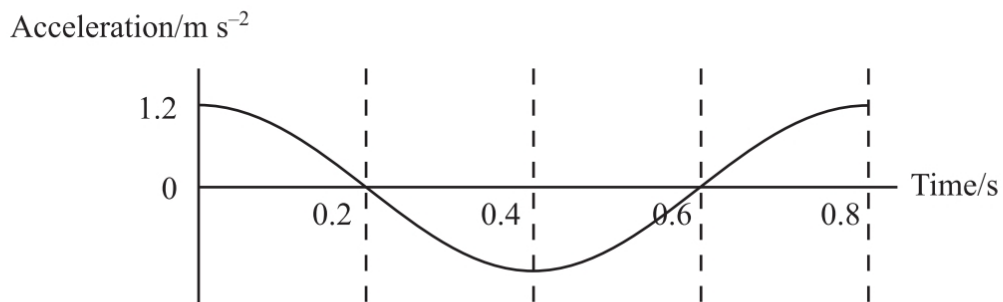
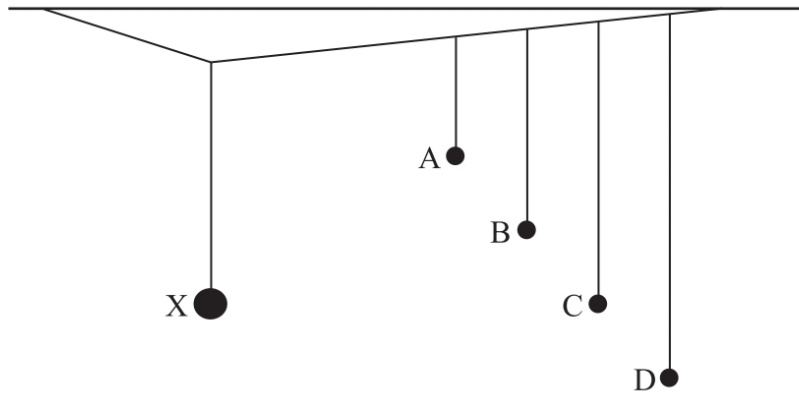Amplitude of oscillation =..

(ii) Sketch a graph of the head's displacement against time over the same time interval on the axes below.

(2)

Displacement

(Total for Question   = 8 marks)

7 The diagram shows a number of pendulums hanging from a single thread. Pendulum X has a heavy lead sphere as the bob and the others have low mass bobs. When X is set into motion energy is transferred to the others which all begin to oscillate.

After a short time C is observed to have the largest amplitude of oscillation.

(a) Explain why pendulum C has the largest amplitude of oscillation.

(3)

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(b) For an efficient energy transfer pendulum C must be at rest when pendulum X has its maximum kinetic energy. The graph below shows how the displacement of pendulum X varies with time.
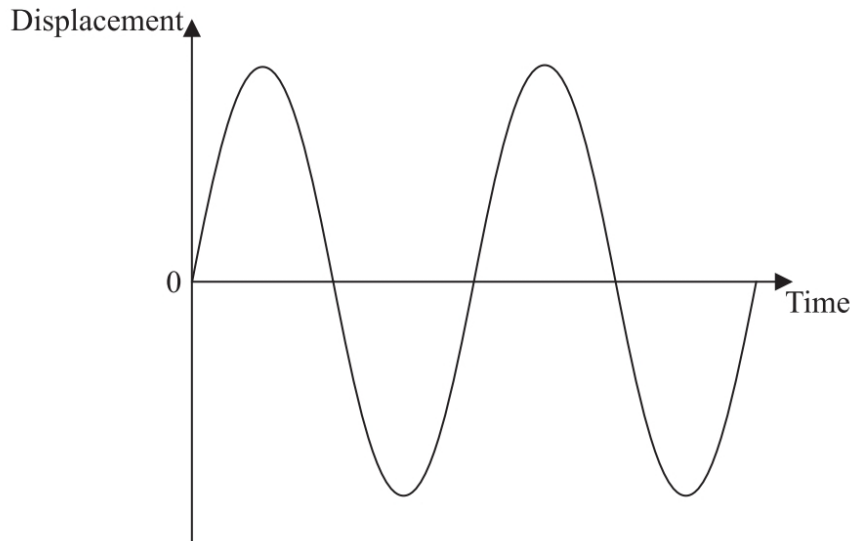
Mark a point P on this graph showing an instant when pendulum X has a maximum kinetic energy, and add a curve to show how the displacement of pendulum C varies over the same time interval.

(2)

(Total for Question   = 5 marks)

8 A baby-bouncer is a light harness, into which a baby can be placed, suspended by a vertical spring.

The height of the baby-bouncer is adjusted so that the baby's feet are a few centimetres above the floor when the baby is in equilibrium in the harness. If the baby is then displaced downwards and released, the system oscillates vertically with simple harmonic motion.

It is stated in a textbook that "a mass-spring system that obeys Hooke's law will lead to simple harmonic motion when the mass is displaced."

*(a) Explain why a system consisting of a mass and a spring that obeys Hooke's law may be set into simple harmonic motion.

(3)

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(b) The acceleration experienced by a baby of mass 8.2 kg is 0.49 m s^{-2} when the displacement from the equilibrium position is 3.0 cm.

Show that the period of vertical oscillations for this baby is about 1.6 s.

(3)

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(c) The amplitude of the oscillations quickly decreases, so the baby has to keep kicking on the floor to maintain them.

(i) State the name given to oscillations that die away quickly.

(1)

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(ii) State the name that is given to oscillations such as those that are kept going by the baby kicking on the floor.

(1)

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(iii) If the baby kicks on the floor at a certain frequency, the amplitude of the bounces can be made to increase to a maximum.

Name this effect and calculate the frequency at which it occurs.

(2)

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Frequency = ...

(d) The baby is replaced by a baby of less mass. This baby also kicks to produce maximum amplitude of oscillation. Without further calculation, explain how the frequency at which the baby must kick compares to that for the larger mass baby.

(2)

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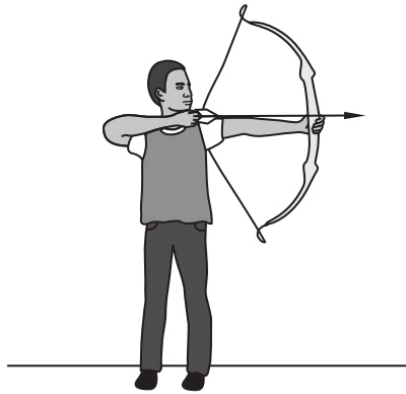
(Total for Question   = 12 marks)

9 An archer is carrying out some target practice with his bow and arrow.

He attaches an apple to a spring hung from a fixed support and sets the apple into vertical oscillation of amplitude 10 cm. The apple performs simple harmonic motion with a frequency of 0.625 Hz.

(a) Describe the conditions required for an oscillation to be simple harmonic.

(2)

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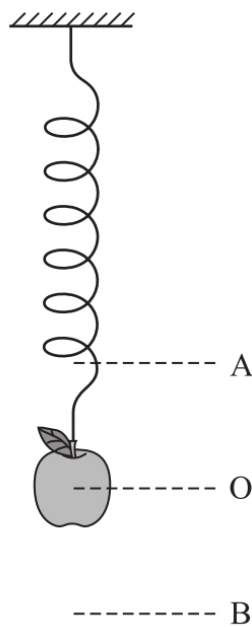
(b) The diagram shows the apple on the spring. A and B are the positions of maximum displacement and O is the equilibrium position of the apple.

Sketch a graph to show how the displacement of the apple varies with time.

(4)

(c) Calculate the maximum velocity of the apple as it oscillates.

(3)

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Maximum velocity = ...

(d) The archer fires an arrow towards the apple as it is oscillating.

Explain at which position of the apple the archer has the best chance of scoring a direct hit.

(2)

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*(e) Over time the amplitude of the apple's oscillation will decrease to zero.

Explain how the principle of conservation of energy applies to this situation.

(3)

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(Total for Question   = 14 marks)